Computational Methods for Identification, Optimization and Control of PDE Systems

John. A. Burns, Eugene. M. Cliff, Lizette Zietsman
Interdisciplinary Center for Applied Mathematics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061-0531

This final report contains a summary of the activities supported under the Air Force AFOSR Grant FA9550-07-1-0273 during the period 1 April 2007 through 30 November 2009. The research focused on the construction of high fidelity numerical methods and the development of a rigorous mathematical framework for attacking complex optimization and control problems with partial differential equations (PDEs) as constraints. The research is motivated by applications to two distinct but related application areas: (1) Optimal design and control of ultra-light large space structures which will serve as the platforms for many future space applications such as space-based radar; (2) Optimal design and control of flexible air vehicles and feedback control of fundamental fluid flows. We generated new numerical methods specifically for parameter estimation, shape optimization, optimal control and feedback control of PDE systems of the type that govern ultra-light inflatable space structures, aero-dynamic design and fluid flows. We developed a rigorous mathematical framework to analyze convergence of the resulting control, design and optimization algorithms, obtain error estimates and rates of convergence.

Introduction and General Goals of the Research Program

The research program is focused on the development of numerical methods and software specifically for the purpose of solving control, design, and optimization problems where the underlying dynamics are described by partial and partial-functional differential equations. Although simulation tools and the numerical methods that provide the foundations of simulation software must play an important role in any research of this type, the demands placed on numerical methods developed specifically to solve control, design, and optimization problems are often different and more complex in nature. With this in mind, we focused on the following general objectives.

- We conducted research on new numerical methods specifically for parameter estimation, shape optimization, optimal control and feedback control of PDE

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14. ABSTRACT
The research focused on the construction of high fidelity numerical methods and the development of a rigorous mathematical framework for attacking complex optimization and control problems with partial differential equations (PDEs) as constraints. The research is motivated by applications to two distinct but related application areas: (1) Optimal design and control of ultra-light large space structures which will serve as the platforms for many future space applications such as space-based radar; (2) Optimal design and control of flexible air vehicles and feedback control of fundamental fluid flows. We generated new numerical methods specifically for parameter estimation, shape optimization, optimal control and feedback control of PDE systems of the type that govern ultra-light inflatable space structures, aero-dynamic design and fluid flows. We developed a rigorous mathematical framework to analyze convergence of the resulting control, design and optimization algorithms, obtain error estimates and rates of convergence.

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systems of the type that govern ultra-light inflatable space structures and specific fluid flows. These numerical schemes will be sufficiently general so as to be applicable to a broad range of Air Force problems.

- We developed a rigorous mathematical framework to analyze convergence of the resulting control, design and optimization algorithms, obtain error estimates and rates of convergence. This framework is essential to the verification of the mathematical design models and corresponding numerical algorithms.

- We worked develop high fidelity and high level computational tools based on the new numerical methods that are suitable for use by engineers working in the design and optimization of structures and fluids. In addition, we investigated a new computational approach to the design of experiments based on bounded error methods.

Accomplishments and Final Report on the Grant

Through the efforts funded under this grant we accomplished several important goals.

- We have shown that continuous sensitivity equations can be used to analyze the impact of un-modeled parameters on closed-loop simulations. Also, we employ the corresponding discrete sensitivity equations to provide information on computational uncertainty with respect to grids and un-modeled parameters. We defined a new "Lyapunov Exponent" in terms of the the system parameters and use this new concept to determine a prediction horizon where qualitative and numerical predictions may be invalid beyond this horizon. We employed this idea to develop insight into the validity of long term simulations. This is particularly important for closed-loop simulations.

- We developed a new computational approach for optimally placing sensor / actuator pairs in order to maximize the controllability and observability of the reduced order approximations of the PDE control systems. We developed a method to study controllability and conditioning of POD models of PDE systems. This work is provided the framework for addressing optimal sensor / actuator location problems.

- We established that the reduced order models must be constructed so that the numerical conditioning of the key control equations (Ricatti, Liapunov, etc.) is as small as possible. We also, discovered new fundamental relationships between the concepts of distance to the nearest uncontrollable system and the conditioning of the approximate Riccati equation for different reduced order models.

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- We developed a new reduced basis construction method which allows for separate consideration of baseline and actuated dynamics as opposed to the case where the baseline and actuated dynamics are considered simultaneously. These different bases give rise to control modeld with varying numerical properties that allow for efficient controller design.

- Developed a new Mesh Independence Principle for PDE Riccati equations and used this method to solve large scale flow control problems.

- Developed well - posed high fidelity model of flexible space structure to support AFRL/VSSV work on ISAT and solved the identification problem for this application

- We developed a new parameter estimation scheme for the design of experiments when insufficient data is available to allow for statistical verification. The method is based on bounded error techniques, but reduces the computational effort by solving an inverse "data-to-parameter" problem. This new approach to parameter estimation and validation that uses the sensitivity methods also developed under this grant and implicit function theorems to map sets in the model's output to the boundary of the parameter membership set.

- We began the work to extend our research on optimal sensor/actuator location to problems with dynamic sensor networks. This effort was motivated by the potential use of MAV based sensors to for information gathering and battlefield management. This work allowed for the inclusion of the MAV sensor platform dynamics.

- We developed an infinite dimensional gradient based algorithm to solve the optimal control problem for the optimal sensor platform control problem with dynamic sensor networks.

In addition, during the grant period we have:

- Produced more than 26 scientific papers,

- Made more than 39 presentations at conferences and colloquium,

- Directed more than 8 graduate students and 1 postdoc,

- Worked with more than 14 visitors, representing 5 different countries,

- Made visits to AFRL at Wright-Patterson and Kirtland AFB to work with Air Force personnel.

Personnel Supported

During this perion, the following people were supported in part under Grant FA9550-07-1-0273:

Senior Investigators	Postdoctoral Fellows	Graduate Students
John A. Burns	Imran Akhtar	Ermira Cami
Eugene M. Cliff		Adam Childers
Lizette Zietsman		Weiwei Hu
Jeff Borggaard		Betty Paredez-Alvarez
		Carlos Rautenberg
		Miroslav Stoyanov
		Golnar Newbury
		Dan Sutton

Participation and Presentations at Meetings 2007-2008

John Burns

1. International Conference on Mathematical Modeling and Computational methods in Science and Engineering, Kobe, Japan, December, 2007.

2. 46th IEEE Conference on Decision and Control, New Orleans, LA, December 2007.

3. The of Alabama Huntsville Distinguished Lecture in Applied Mathematics, Huntsville, AL, November, 2008.

4. Atlantic Coast Symposium on the Mathematical Sciences in Biology and Medicine, Raleigh, NC, April, 2008.

5. Workshop on High Performance Buildings, Berkeley, CA, May, 2008.

6. United Technology – LBNL Workshop on High Performance Buildings, June, 2008.

7. AFOSR Conference on Control, Arlington, VA, August, 2008.

8. International Conference on Shape and Topology Optimization, Graz, Austria, September, 2008.

9. Alabama Huntsville Distinguished Lecture in Applied Mathematics, Huntsville, AL, November, 2008.

10. Computational Issues in High Performance Buildings, ORNL, Oak Ridge, TN, December, 2008.

11. 47th IEEE Conference on Decision and Control, New Orleans, LA, December, 2008.

12. Workshop on PDE Optimization, Oberwolfach, Germany, January, 2009.

13. International Conference on Approximation Methods for Design and Control, Universidad Tecnologica Nacional, Buenos Aires, Argentina, March, 2009.

14. Conference on Nonlinear Analysis, Donghua University, Shanghai, China, May 2009.

15. Conference on Mathematical Control Theory, Beijing, China, May 2009.

16. American Control Conference, St. Louis, Missouri, June, 2009.

17. 24th IFIP TC7 Conference on Systems and Control, Buenos Aires, Argentina, July, 2009.

18. Conference on Control, Arlington, VA, July, 2009.

19. Conference on Mathematical Methods and Modeling in Life Sciences and Biomedicine, Sile, Turkey, August, 2009.

20. Conference on Systems Theory, KTH, Stockholm, Sweden, September, 2009.

21. Virginia Governor's School, Pulaski, VA, September, 2009.

22. University of Minnesota, Minneapolis, MN, October, 2009.

23. Second International Conference on Mathematical Modeling and Analysis of Populations in Biological Systems, Huntsville, Alabama, October, 2009.

24. Issues in Nonlinear Control, Monterey, California, November 2009.

Gene Cliff

1. Radiative Cooling of an Inflated Longeron, 7th International Conference on Computational and Mathematical Methods in Science and Engineering, Chicago, IL, June 2007.

2. Thermal Issues in Deployment of an Inflatable/Rigidizable Space Structure, XVI Congresso sobre Metodos Numericos y sus Aplicaciones, Ciudad de Cordoba - Argentina, October 2007.

3. Inverse Problems in Deployable Space Structures, Fourth Conference on Inverse Problems: Modeling and Simulation, Fethiye, Turkey, May 2008.

Lizette Zietsman

1. Inverse Problems: Modeling and Simulation, \"Ol\"udeniz, Fethiye, Turkey, May, 2008.

2. Mathematical Theory of Networks and Systems, Blacksburg, VA, July, 2009.

3. AFOSR: Computational Mathematics, Arlington, VA, August, 2008.

4. 4th AIAA Flow Control Conference, Seattle, WA, June, 2008

5. Colloquium: Department of Scientific Computing, Florida State, FL, October, 2008

6. 47th IEEE Conference on Decision and Control, Cancun, Mexico, December.

7. SIAM Conference on Computational Science and Engineering (CSE09), Miami, FL, March, 2009.

8. International Conference on Approximation Methods for Design and Control, December, 2009.

9. International Conference on Approximation Methods for Design and Control, Universidad Tecnologica Nacional, Buenos Aires, Argentina, March, 2009.

10. SIAM Conference on Control and Its Applications, Denver, Colorado, July, 2009.

11. SIAM Annual Meeting, Denver, Colorado, July, 2009.

12. Department of Mathematics,University of Pittsburgh, Pittsburgh, PA, November, 2009.

Publications Supported Under this Grant for 2007-2009

1. J. A. Burns, E. M. Cliff, Z. Liu and R. D. Spies, Stability of a Joint-Leg-Beam System with Local Damping, *Journal of Mathematical and Computer Modeling*, 46 (2007) 1236–1246.

2. J. A. Burns and L. Zietsman, Upwind Approximations and Mesh Independence for LQR Control of Convection Diffusion Equations, *46th IEEE Conference on Decision and Control*, 2007, 219–224.

3. J. A. Burns, E. M. Cliff and S. Doughty, Analysis and Parameter Estimation for a Model of Chlamydia Trachomatis Infection, *J. Inverse and Ill Posed Problems*, 15 (2007), 243-256.

4. E. M. Cliff, T. L. Herdman, and Z. Y. Liu, Angular-Impulse Control for a Large Flexible Spacecraft, *J. of Guidance, Control, and Dynamics*, 30 (2007), 87-99.

5. Y.Lei, C. Cao, E.M. CLiff, N. Hovakimyan, and A. Kurdila, Design of an L1 Adaptive Control for an air-breathing hypersonic vehicle in the presence of unmodeled dynamics, *AIAA-2007-6527*, August. 2007.

6. Lizette Zietsman, A Numerical Study of Controllability and Conditioning of POD Models, *Proceedings of the 4th AIAA Flow Control Conference, Seattle, Washington*, (2008).

7. J. A. Burns, E. W. Sachs and Lizette Zietsman, Mesh Independence of Kleinmann-Newton Iterations for Riccati Equations on Hilbert Space, *SIAM Journal on Control and Optimization*, Vol. 47, No. 5, (2008), pp. 2663—2692.

8. A. J. van der Merwe, N. F. J. van Rensburg and Lizette Zietsman, Solvability of a Reissner-Mindlin-Timoshenko plate-beam vibration model, *IMA Journal of Applied Mathematics*, (2009), Vol 74, No. 1, 149–162.

9. K. A. Evans and Lizette Zietsman, Riccati Conditioning and Sensitivity for a MinMax Controlled Cable-Mass System, *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, (2008),pages, 4007–4011.

10. D. Pelletier, A. Hay, S. Etienne and J. Borggaard, The Sensitivity Equation Method in Fluid Mechanics, *European Journal of Computational Mechanics*, Vol. 17, No. 1–2, pages 31–61 (2008).

11. J. Borggaard, T. Iliescu, H. Lee, J.-P. Roop and H. Son, A Two-Level Discretization Method for the Smagorinsky Model, *SIAM Journal of Multiscale Modeling and Simulations*, Vol. 7, No. 2, pages 599–621 (2008).

12. I. Akhtar, J. Borggaard and J. Burns, Reduced-Order Models for Optimal Control of Vortex Shedding, *Proceedings of the 4th AIAA Flow Control Conference*, AIAA Paper Number 2008-4083 (2008).

13. I. Akhtar, J. Borggaard, J. Burns and L. Zietsman, Model-Based Computation of Functional Gains for Feedback Control of Vortex Shedding, in *Proceedings of the 2008 ASME International Mechanical Engineering Congress & Exposition*, Paper Number IMECE2008-68950 (2008).

14. J. Borggaard and M. Stoyanov, An Efficient Long-Time Integrator for Chandrasekhar Equations, *Proceedings of the 47th IEEE Conference on Decision and Control*, Paper Number ThTA07.1 Cancun, Mexico, (2008).

15. J. Borggaard and M. Stoyanov, A Reduced Order Solver for Lyapunov Equations with High Rank Matrices, *Proceedings of the 18th International Symposium on Mathematical Theory of Networks and Systems*, Blacksburg, VA (2008).

16. J.A. Burns, E.M. Cliff, Z. Y. Liu, and R. Spies, On Coupled Transversal and Axial Motions of Two Beams with a Joint, *J. Math. Anal. Appl.* 339 (2008), no. 1, 182–196.

17. E.M. Cliff, R.J. Kraus, J.C. Luby and C.A. Woolsey, Optimal Control of an Undersea Glider in a Symmetric Pull-Up, *International Symposium on the Mathematical Theory of Networks and Systems* (MTNS), Blacksburg, VA, 28 July - 1 August 2008.

18. E.M. CLiff, S. A. Lane and T.W. Murphey, A. Lane Deriving Space Antenna Structural Architectures from Electronic Deformation Compensation Strategies, *IEEE Trans. on Aerospace and Electronic Systems*, submitted.

19. John A. Burns and Lizette Zietsman, Upwind Approximations and Mesh Independence for LQR Control of Convection Diffusion Equations, 46th *IEEE Conference on Decision and Control*, New Orleans, December 2007.

20. John A. Burns and Lisa Davis, Sensitivity Analysis and Computational Uncertainty with Applications to Control of Nonlinear Parabolic Partial Differential Equations, 47th *IEEE Conference on Decision and Control*, Cancun, Mexico, December 2008, 3989 - 3994.

21. E. Allen, J.A. Burns and D. Gilliam, Control and Bifurcation Under Uncertainty of Nonlinear Convection-Diffusion Equations, 47th *IEEE Conference on Decision and Control*, Cancun, Mexico, December 2008, 197 – 202.

22. J. Borggaard, J.A. Burns, A. Surana and L. Zietsman, Control, Estimation and Optimization of Energy Efficient Buildings, *2009 American Control Conference*, Paper Number WeB05.4 (2009).

23. J. A. Burns, E. M. Cliff and C. N. Rautenberg, A Distributed Parameter Control Approach to Optimal Filtering and Smoothing with Mobile Sensor Networks, 17th *Mediterranean Conference on Control and Automation*, June 2009, 181–186.

24. J. A. Burns and Adam Childers, Bounded Error Parameter Estimation for Non-Linear Continuous Dynamical Systems, 17th *Mediterranean Conference on Control and Automation*, June 2009, 193–198.

25. Y.Lei, C. Cao, E.M. CLiff, N. Hovakimyan, A. Kurdila, and K. Wise, Design of an L1 Adaptive Controller for a Hypersonic Vehicle with Flexible Body Dynamics, Paper ThB16.4, *American Control Conference*, St Louis, MO, June 2009.

26. J. Borggaard, J. A. Burns, E. M. Cliff and L. Zietsman, PDE Approach to Optimization and Control of High Performance Buildings, in *Proceedings of the Oberwolfach Workshop on Numerical Techniques for Optimization Problems with PDE Constraints*, M. Heinkenschloss, R. H. W. Hoppe and V. Schulz, Eds., January 2009, 205–208

Honors & Awards Received

- Mr. Carlos Rautenberg: The Lee and Regina Steeneck Scholarship Prize for Excellence in Research

AFRL and DOD Points of Contact

- Dr. Siva Banda, Control Sciences Center of Excellence, AFRL/VACA, WPAFB, (937) 255-8676.

- Dr. Thomas Murphey, ISAT IPT Lead, AFRL/VSSV, Kirtland AFB, New Mexico, (505) 846-9969.